BORE HOLE TOOL ASSEMBLY, BEARING SYSTEM FOR USE IN SUCH A
BORE HOLE TOOL ASSEMBLY, AND METHOD OF DESIGNING SUCH A
BORE HOLE TOOL ASSEMBLY

The present invention relates to a bore hole tool
assembly, in particular for use in a hydrocarbon well
bore hole, comprising a housing and a mandrel reaching
along an axial direction into the housing and a bearing
5   system for transferring an axial load between the housing
and the mandrel and allowing rotation of the housing
relative to and about the mandrel, wherein the bearing
system comprises at least two bearing stages each
arranged to transfer part of the axial load, and each
10   bearing stage comprising bearing means.
In further aspects, the invention relates to a
bearing system for use in such a bore hole tool assembly,
and to a method of designing such a bore hole tool
assembly.
15   In oil well drilling and completion operations, a
bore hole tool assembly in the form of a tubular drill
string is used. These drill strings can consist of
various types and sizes of tubular components like drill
pipes, heavy weight drill pipe, drill collars, liner,
20   casing and tubing. These components can be used in
various configurations.
These drill strings are subjected to mechanical
tension and compression due to the weight of the drill
string and friction against the well bore wall. A bearing
25   assembly is required, if a portion of the drill string is
to be independently rotated from the remaining part of
the drill string. Such bearing assembly is subjected to
the same tensile and compressive loading as the
surrounding tubular members in the drill string.

In this type of use, the bearing assembly generally
is provided in the annular space that is available
between a tubular end of one drill string component,
functioning as the mandrel, and a tubular end of another
drill string component, functioning as the housing.

The cross sections of the mandrel and that of the
housing are often constrained as a consequence of
practical requirements. For instance, the maximum cross
section of tubulars for use in a well bore is limited by
the need of an annular space between the well bore and
the tubulars for allowing sufficient flow of a drilling
fluid containing drilled cuttings. The minimum cross
section may be constrained by the demand for allowing
sufficient flow of the drilling fluid or for enabling
passage of objects of a given size such as cementation
plugs or wire-line deployed tools. In addition, a minimum
wall thickness of mandrel and housing are required for
providing sufficient strength.

These requirements limit the space available for
mounting the bearing assembly in the annular space. A
single bearing stage that fits in the available space may
have insufficient strength to bear the load that the bore
hole tool assembly is subjected to. Therefore, it has
been proposed to use a bearing system having a plurality
of bearing stages arranged to transfer part of the axial
load parallel to each other.

In practice, the respective bearing stages are
arranged in the annular space with an axial displacement
relative to each other, such that they act parallel to
each other in transferring the axial load from the
housing to the mandrel.

It has been found that, where each bearing stage of
the bearing system can transfer a certain maximum static
load before failure, the bearing series already fails
under a static load that is significantly lower than the

mentioned maximum load per bearing stage times the number
of bearing stages in the series. Moreover, under rotary
conditions the endurance life of the system is
significantly less that would be expected from the
5   specified endurance life of the individual bearings.

It is thus an object of the invention to increase the
maximum allowable load for the bore hole tool assembly,
and in particular for the bearing system.

It is another object of the invention to provide a
10   bore hole tool assembly, and a bearing system for such a
bore hole tool assembly, having a longer operational
lifetime.

In accordance with the invention, one or more of
these objects are achieved in a bore hole tool assembly
15   comprising a housing and a mandrel reaching along an
axial direction into the housing and a bearing system for
transferring an axial load between the housing and the
mandrel and allowing rotation of the housing relative to
and about the mandrel, wherein the bearing system
20   comprises at least two bearing stages each arranged to
transfer part of the axial load, whereby each bearing
stage comprises bearing means and mitigation means for
distributing the load substantially proportionally over
the respective bearing stages.

25   The invention is based on the insight that the
mandrel and the housing in practice each behave as
elastically deformable members having a certain
mechanical stiffness. The entity mechanical stiffness is
for the purpose of the present description defined as the
30   ratio of the force transmitted through a member over a
certain length and the amount of increment or decrement
of that length. In order to avert an uneven distribution
of the load over the bearing stages resulting from the
elastic behaviour of the mandrel, the housing and the
35   bearings, the mitigation means serves to distribute the

load substantially proportionally over the respective
bearing stages. Thereby it is substantially avoided that
some of the bearing stages are loaded beyond their
maximum allowable load while other bearing stages in the
series could still have taken the excess portion of the
load.

A substantially proportional distribution of the
total load over the bearing stages is for the purpose of
this patent application understood to include a
distribution whereby the load on the bearing stage that
receives the highest fraction of the total load does not
exceed 1.5 times the average load being the total load
divided by the number of bearing stages in the bearing
system. Preferably it does not exceed 1.3 times the
average load, and more preferably it does not exceed 1.1
times the average load.

For avoiding prematurely exceeding the maximum load
in one of the bearing stages, it is sufficient that the
mitigation means are arranged to distribute the load
substantially proportionally over the respective bearing
stages only when the load on one or more of the bearing
stages approaches the maximum allowable load.

Since the entire bearing system fails upon failure of
only one bearing stage, also the operational lifetime is
increased as a result of the mitigation of the load from
the most heavy-loaded bearing stage.

In an embodiment, the mitigation means comprises
deformable means arranged to transfer at least part of
the axial load in series with the respective bearing
means. Deformable means deform under a load, so that the
load in the bearing stage that takes up more than its
proportional share is partially mitigated into
deformation of the deformable means whereby consequently
the remaining bearing stages are forced to pick up a
relatively higher portion of the load as a result.

The deformable means may comprise a network of flexible fluid bellows that are in mutual fluid communication, whereby each bearing stage comprises such a flexible fluid bellow. Alternatively, deformable means may be made of combined elastic and plastic deformable material, or of essentially elastic deformable material.

An advantage of having elastic or elastic and plastic deformable mitigation means is that this allows for elastically biasing the mitigation means against the bearing means which therefore remain operative over a range of loads including medium high loads compared to the maximum allowable load. When also medium high loads are distributed substantially proportionally over the bearing stages, the wear on the bearing means is distributed more evenly over the respective bearing stages and thus the overall lifetime of the entire bearing system is further improved.

Preferably the mechanical stiffness of the deformation means in at least one of the stages is lower than that of the housing and that of the mandrel bridged by that stage. Incompatibility between the elastic behaviour in the housing and the mandrel is herewith suitably compensatable.

Ideally, the axial load is most easily distributed proportionally over the available bearing stages if the mechanical stiffness of each bearing stage is lower than the mechanical stiffness of the housing and than that of the mandrel by at least about 2 to 2.5 times per bearing stage in the system. However, this would require a long axial length for the entire bearing system, which is not normally not available nor desirable.

For this reason, the mechanical stiffness of the deformable means in one bearing stage is preferably different in magnitude from that of the deformable means in the other bearing stage. Herewith the axial load

transferred by each bearing stage can be re-distributed
over the bearing stages as desired, since the relative
axial load taken by the bearing stage having the lower
mechanical stiffness is lower than it would have been

5   having equal mechanical stiffness as the other bearing
stage. Thus by providing the lowest mechanical stiffness
in the bearing stage that is otherwise the most heavy
loaded, the load is distributed more proportionally over
the bearing stages.

10   Herewith the axial length required for the bearing
system is reduced. In practice, the mechanical stiffness
in the bearing stages can be higher than one tenth of
that the housing and/or the mandrel.

        According to the second aspect of the invention there

15   is provided a bearing system, for use in a bore hole tool
assembly, in particular for an oil field tool assembly,
for transferring an axial load between a housing and a
mandrel reaching along the axial direction into the
housing, and allowing rotation of the housing relative to

20   and about the mandrel, which bearing system comprises at
least two bearing stages each arranged to transfer part
of the axial load, whereby each bearing stage comprises
bearing means and mitigation means for distributing the
load substantially proportionally over the respective

25   bearing stages.

        Advantages and further embodiments of the bearing
system according to the invention have been presented in
the description relating to the bore hole tool assembly
above.

30   In accordance with the third aspect of the invention,
there is provided a method of designing a bore hole tool
assembly in accordance with the description above, the
method comprising the steps of:
        a)   selecting a first bearing stage stiffness value for

35   one bearing stage;

b)   calculating, for given stiffness values of the parts
of the housing that stretch between the bearing stages,
and given stiffness values of the parts of the mandrel
that stretch between the bearing stages, the value for
the second bearing stage stiffness whereby the axial load
is distributed substantially proportionally over the
respective bearing stages, thereby using the selected
first bearing stage stiffness value as input.

Herewith, a troublesome and time-consuming empirical
try-and-error routine on a prototype model is avoidable.

This method can be part of a method of producing the
bore hole tool assembly, wherein housing, mandrel, and
bearing system comprising the bearing stages in
accordance with the calculation are selected and
assembled.

In particular where both the mandrel and the housing
have a mechanical stiffness that is constant along the
bearing stages, the appropriate values for the mechanical
stiffness of the remaining bearing stages are
advantageously expressed in stiffness ratios between the
first bearing stage stiffness and the stiffness of the
housing, and between the first bearing stage stiffness
and the stiffness of the mandrel.

The invention will described hereinafter in more
detail and by way of example, with reference to the
accompanying drawings in which:

Fig. 1 schematically shows a cross sectional view of
a bore hole tool assembly;

Fig. 2 shows an elastic-mechanical representation of
the bore hole tool assembly shown in Fig. 1 for the case
of tension (part a) and compression (part b).

In the Figures like reference signs relate to like
components.

Referring to Fig. 1 there is schematically shown a
bore hole tool assembly 1 in cross sectional view. The

bore hole tool assembly comprises a tubular housing 2,
and a tubular mandrel 3 reaching axially into the
housing 2, leaving an annular space between the mandrel 3
and the housing 2. A number of bearing stages (8,9) is

5   provided in the annular space, whereby each bearing stage
connects to the mandrel on one side and to the housing on
the other side. The bearing stages 8 are arranged to
transmit, parallel to each other, a tensile axial force
between the mandrel 3 and the housing 2; the bearing

10   stages 9 are arranged to transmit a compressive force
between the housing 2 and the mandrel 3.

Still referring to Fig. 1, the bearing stages (8,9)
each comprise bearing means (4,6) and mitigation
means (5,7). The bearings may be of any type, including

15   roller bearings, ball bearings, spherical roller
bearings, tapered roller bearings, frusto-conical roller
bearings. Elongate bearings, radially arranged with
respect to the axis of rotation of the housing 2 and
mandrel 3, are preferred because the load in such

20   bearings is distributed over a larger part of the bearing
surface. The mitigation means can be provided in the form
of any deformable material, such as a plastic or rubber
like material, or in the form of a compressive spring
such as an annular disc spring. As can be seen, the

25   mitigation means is provided to transmit the load in each
bearing stage in series with the respective bearing means
in that stage.

By providing the deformable material in series with
the bearing means, the mechanical stiffness of the

30   bearing stage is lowered with respect to the mechanical
stiffness of the bearing means per se. The distribution
of the force over the available bearing stages is
controllable by providing a bearing stage that in absence
of the mitigation means would pick up a load higher than

35   proportional with a lower mechanical stiffness than a

bearing stage that in absence of the mitigation means
would pick up a load lower than proportional.

The bore hole tool assembly depicted in Fig. 1 has a
bearing system with four bearing stages. It has been
found that for use in a typical hydrocarbon well bore
four bearing stages provides a suitable optimisation
between strength, reliability, endurance life, and size
of the assembly. However, it will be appreciated that the
number of bearing stages can be varied to any number of
bearing stages with a minimum of two.

Below, the method for designing the bore hole tool
assembly in accordance with the invention is further
elucidated with reference to a mathematical approach. The
mathematical approach is generalised for any number of
bearing stages, denoted by n. It is remarked that the
results of the mathematical approach discussed below hold
for both compressive and tensile loading, and it also
holds for a case wherein the consecutive deformable means
are pre-strained against each other.

Fig. 2 shows an elastic-mechanical representation of
the bore hole tool assembly that is depicted in Fig. 1,
whereby Fig. 2a relates to the bore hole tool under a
tensile load and Fig. 2b relates to the bore hole tool
under a compressive load. Each of the bearing stages is
numbered with the index k, whereby k = 1, 2, ..., n. The
index k = 1 refers to the first bearing stage where the
full load (tension or compression) acts in the mandrel.
The index k = n refers to the last bearing stage where
the full load (tension or compression) acts in the
housing.

The housing and mandrel may consist of different
sections, having for instance different lengths or
different cross sectional areas. Each such section is
denoted by an index i. The total number of sections is
2n. Each such section has a mechanical stiffness, defined

as the ratio of the force transmitted through the section
having a certain length and the amount of increment or
decrement of that length. The stiffness values of the
respective sections along the housing are denoted by K_{hi},
whereby i = 1, 2, ..., 2n. The stiffness values of the
respective sections along the mandrel are denoted by
K_{mi} i = 1, 2,..., 2n.

Displacements of indexed points between the sections
along the mandrel and housing are given by U_j, whereby
j = 1, 2, 3, ..., (4n+2). Even numbers j correspond to
points on the housing and odd numbers j correspond to
points on the mandrel.

The mandrel stiffness K_{mi} and housing stiffness K_{hi}
for a certain section (with number i) can be calculated
on the basis of the dimensions and material properties of
the cross sections using the following equations:

$$K_{hi} = \frac{A_{hi}E_h}{L_i},$$

$$K_{mi} = \frac{A_{mi}E_m}{L_i}$$

wherein

A_{hi} is the cross sectional area of housing section i
[m^2];

A_{mi} is the cross sectional area of mandrel section i
[m^2];

L_i is the length of housing or mandrel section i [m];

E_h is the elasticity modulus of housing material [N/m^2];
and

E_m is the elasticity modulus of mandrel material [N/m^2].

The bearing stage stiffnesses K_1, K_2, ..., K_n
correspond to the combined stiffnesses of the combination
of a bearing and a deformable means, in this example in
the form of a spring, in series in the bearing stage with
number k

$$K_k = \frac{K_{spring,k} K_{bearing,k}}{K_{spring,k} + K_{bearing,k}}$$

wherein $K_{bearing,k}$ and $K_{spring,k}$ respectively are the bearing stiffnesses and the spring stiffness for the k-th bearing stage.

It is a task to find values for the respective spring stiffnesses $K_{spring,k}$ such that the total axial force F, transmitted between the mandrel and the housing, is distributed proportionally over the available bearing stages. In other words, the force F_k transmitted by each bearing stage must ideally be 1/n-th of the total force:

$$F_k = \frac{1}{n} F \ .$$

In a situation where the proportional force distribution is achieved, the displacements U_j (see Fig. 2) along the mandrel become:

$U_1 = 0$, (Taken as a reference)

$$U_3 = \frac{F}{K_{m1}},$$

$$U_5 = U_3 + \frac{n-1}{n} \frac{F}{K_{m2}} = U_3 + \frac{1}{n}\left[(n-1)\frac{1}{K_{m2}}\right]F,$$

$$U_7 = U_5 + \frac{F}{K_{m3}} = U_3 + \frac{1}{n}\left[(n-1)\left(\frac{1}{K_{m2}} + \frac{1}{K_{m3}}\right)\right]F,$$

$$U_9 = U_7 + \left(1 - \frac{2}{n}\right)\frac{F}{K_{m4}} = U_3 + \frac{1}{n}\left[(n-1)\left(\frac{1}{K_{m2}} + \frac{1}{K_{m3}}\right) + (n-2)\left(\frac{1}{K_{m4}}\right)\right]F,$$

$$U_{11} = U_9 + \left(1 - \frac{2}{n}\right)\frac{F}{K_{m5}} = U_3 + \frac{1}{n}\left[(n-1)\left(\frac{1}{K_{m2}} + \frac{1}{K_{m3}}\right) + (n-2)\left(\frac{1}{K_{m4}} + \frac{1}{K_{m5}}\right)\right]F,$$

$$U_{13} = U_{11} + \left(1 - \frac{3}{n}\right)\frac{F}{K_{m6}} = U_3 + \frac{1}{n}\left[(n-1)\left(\frac{1}{K_{m2}} + \frac{1}{K_{m3}}\right) + (n-2)\left(\frac{1}{K_{m4}} + \frac{1}{K_{m5}}\right) + (n-3)\left(\frac{1}{K_{m6}}\right)\right]F,$$

$$U_{15} = U_{13} + \left(1 - \frac{3}{n}\right)\frac{F}{K_{m7}} = U_3 + \frac{1}{n}\left[(n-1)\left(\frac{1}{K_{m2}} + \frac{1}{K_{m3}}\right) + (n-2)\left(\frac{1}{K_{m4}} + \frac{1}{K_{m5}}\right) + (n-3)\left(\frac{1}{K_{m6}} + \frac{1}{K_{m7}}\right)\right]F,$$

etc.

A first bearing stage stiffness value K_1 is selected. Normally, the stiffness of the bearing means has a given value so that the selected bearing stage stiffness is obtained by proper selection of the mitigation means

having the appropriate stiffness value. The selection of
a suitable bearing stage stiffness can based on strength
considerations. In the desired situation of
proportionally distributed force, the displacements U_j
long the housing can be expressed in terms of K_1:

$$U_2 = U_3 + \frac{1}{K_1}\frac{F}{n},$$

$$U_4 = U_2 + \frac{1}{n}\frac{F}{K_{h1}} = U_3 + \frac{1}{n}\left[\frac{1}{K_1} + \frac{1}{K_{h1}}\right]F,$$

$$U_6 = U_4 + \frac{1}{n}\frac{F}{K_{h2}} = U_3 + \frac{1}{n}\left[\frac{1}{K_1} + \frac{1}{K_{h1}} + \frac{1}{K_{h2}}\right]F,$$

$$U_8 = U_6 + \frac{2}{n}\frac{F}{K_{h3}} = U_3 + \frac{1}{n}\left[\frac{1}{K_1} + \frac{1}{K_{h1}} + \frac{1}{K_{h2}} + \frac{2}{K_{h3}}\right]F,$$

$$U_{10} = U_8 + \frac{2}{n}\frac{F}{K_{h4}} = U_3 + \frac{1}{n}\left[\frac{1}{K_1} + \frac{1}{K_{h1}} + \frac{1}{K_{h2}} + \frac{2}{K_{h3}} + \frac{2}{K_{h4}}\right]F,$$

$$U_{12} = U_{10} + \frac{3}{n}\frac{F}{K_{h5}} = U_3 + \frac{1}{n}\left[\frac{1}{K_1} + \frac{1}{K_{h1}} + \frac{1}{K_{h2}} + \frac{2}{K_{h3}} + \frac{2}{K_{h4}} + \frac{3}{K_{h5}}\right]F,$$

$$U_{14} = U_{12} + \frac{3}{n}\frac{F}{K_{h6}} = U_3 + \frac{1}{n}\left[\frac{1}{K_1} + \frac{1}{K_{h1}} + \frac{1}{K_{h2}} + \frac{2}{K_{h3}} + \frac{2}{K_{h4}} + \frac{3}{K_{h5}} + \frac{3}{K_{h6}}\right]F,$$

etc.

Having made the selection of the stiffness value for
K_1, the optimal values for the remaining bearing stage
stiffnesses K_2, K_3,.., K_n can be mathematically solved in
order to obtain the uniform force distribution. This is
done as follows. For the optimal stiffness of the second
bearing stages should hold:

$$F_2 = K_2(U_6 - U_7) \equiv \frac{1}{n}F,$$

$$F_3 = K_3(U_{10} - U_{11}) \equiv \frac{1}{n}F$$

$$F_4 = K_4(U_{14} - U_{15}) \equiv \frac{1}{n}F,$$

etc.

When substituting the expressions given above for the
displacements U_j along the mandrel (j odd) and housing (j
even), this yields for the optimal ratios of the bearing
stage stiffnesses K_k/K_1, k = 2, 3, ..., n:

$$\frac{K_2}{K_1} = \left[1 + \left(\frac{K_1}{K_{h1}} + \frac{K_1}{K_{h2}}\right) - (n-1)\left(\frac{K_1}{K_{m2}} + \frac{K_1}{K_{m3}}\right)\right]^{-1},$$

$$\frac{K_3}{K_1} = \left[1 + \left(\frac{K_1}{K_{h1}} + \frac{K_1}{K_{h2}}\right) + 2\left(\frac{K_1}{K_{h3}} + \frac{K_1}{K_{h4}}\right) - (n-1)\left(\frac{K_1}{K_{m2}} + \frac{K_1}{K_{m3}}\right) - (n-2)\left(\frac{K_1}{K_{m4}} + \frac{K_1}{K_{m5}}\right)\right]^{-1},$$

$$\frac{K_4}{K_1} = \left[\begin{array}{l} 1 + \left(\frac{K_1}{K_{h1}} + \frac{K_1}{K_{h2}}\right) + 2\left(\frac{K_1}{K_{h3}} + \frac{K_1}{K_{h4}}\right) + 3\left(\frac{K_1}{K_{h5}} + \frac{K_1}{K_{h6}}\right) \cdots \\ \cdots - (n-1)\left(\frac{K_1}{K_{m2}} + \frac{K_1}{K_{m3}}\right) - (n-2)\left(\frac{K_1}{K_{m4}} + \frac{K_1}{K_{m5}}\right) - (n-3)\left(\frac{K_1}{K_{m6}} + \frac{K_1}{K_{m7}}\right) \end{array}\right]^{-1},$$

etc.

These equations hold for the general situation where the stiffness values in the housing and the mandrel can be different for each section i. However, for a perhaps more realistic situation where the housing and mandrel
5 stiffnesses are constant along the sections, i.e. assuming that $K_{m1} = K_{m2} = \ldots = K_{mn} \equiv K_m$ for the mandrel and that $K_{h1} = K_{h2} = \ldots = K_{hn} \equiv K_h$ for the housing, the equations can be significantly simplified. In that case the optional bearing stage stiffnesses are given by

$$\frac{K_k}{K_1} = \left[1 + A_k \frac{K_1}{K_h} + B_k \frac{K_1}{K_m}\right]^{-1},$$
$$A_k = k(k-1),$$
$$B_k = (k-1)(k-2n)$$

10 As can be seen, the optimum stiffness ratios K_k/K_1 whereby the force is distributed proportionally over the available bearing stages, depend only on the ratios of K_1/K_h and K_1/K_m. The differences in optimal stiffnesses decrease as the value of the first bearing stage
15 stiffness is selected lower.

It is remarked that the entity stiffness, including K_h, K_m, K_1, K_k is given in dimensions of force per unit distance [N/m]. The factors A_k and B_k, as well as the indexing numbers, are dimensionless.

20 If the mechanical stiffness of the deformable mitigation means in each bearing stage is lower than the mechanical stiffness of the housing or that of the

mandrel (whichever of the two is the lowest) by at least
2 to 2.5 times per bearing stage in the bearing system,
then the axial load is most easily distributed
proportionally over the available bearing stages because
5   the ratios between the various bearing stage stiffnesses
approaches unity.

However, with the constraints imposed in practical
application, such as size and required strength, the
mechanical stiffness of the deformable mitigation means
10   in each bearing stage lies somewhere between 0.25 and
2.5 times per bearing stage lower than the lower value of
the stiffness of the housing or mandrel. In that case, a
more proportional distribution of the force over the
available bearing stages is obtainable by optimisation of
15   the respective bearing stage stiffnesses. Preferably, it
is herewith prevented that one single bearing stage takes
a significantly larger than proportional part of the
total load, for instance at most 1.5 × F/n.

For a bore hole tool assembly having a bearing system
20   with two bearing stages, and a constant stiffness along
the mandrel and the housing, the optimal stiffness K_2
that matches with K_1 as selected to achieve a
proportional force distribution over both bearing stages
is thus given by:

$$\frac{K_2}{K_1} = \left[1 + 2\frac{K_1}{K_h} - 2\frac{K_1}{K_m}\right]^{-1}$$

25   For a bore hole tool assembly having a bearing system
with three bearing stages, and a constant stiffness along
the mandrel and the housing, the optimal stiffnesses K_2
and K_3 are:

$$\frac{K_2}{K_1} = \left[1 + 2\frac{K_1}{K_h} - 4\frac{K_1}{K_m}\right]^{-1},$$

$$\frac{K_3}{K_1} = \left[1 + 6\frac{K_1}{K_h} - 6\frac{K_1}{K_m}\right]^{-1}$$

For a bore hole tool assembly having a bearing system
with four bearing stages, and a constant stiffness along
the mandrel and the housing, the optimal stiffnesses K_2,
K_3 and K_4 are:

$$\frac{K_2}{K_1} = \left[1 + 2\frac{K_1}{K_h} - 6\frac{K_1}{K_m}\right]^{-1},$$

$$\frac{K_3}{K_1} = \left[1 + 6\frac{K_1}{K_h} - 10\frac{K_1}{K_m}\right]^{-1},$$

$$\frac{K_4}{K_1} = \left[1 + 12\frac{K_1}{K_h} - 12\frac{K_1}{K_m}\right]^{-1}$$

5   It has been assumed in the above explanation that the
mechanical stiffness behaves linear with the load. In
cases where the mechanical response of a member to a load
is non-linear, the mechanical stiffness of that member
should be determined for the load that occurs if the
10   total design load for the bearing system is sufficiently
proportionally distributed over the bearing stages.

It will be appreciated that practical constraints
such as availability of springs can lead to a design
where some of the section lengths L_i of the housing/
15   mandrel need to be longer than others in order to obtain
a desired bearing stage stiffness. The equations given
above must then be adapted for that specific case.

Example

An oil well drilling tool which is capable of
20   transferring 2000 kN both in tensile as well as in
compressive loading is considered. The total force needs
to be equally distributed over n = 4 bearing stages. In
this way the static strength of any of the bearings is
not exceeded prematurely and in addition the endurance of
25   the bearing system life is improved. The tool is based on
a circular cylindrical mandrel and a circular cylindrical
housing.

The functional specifications for the design put
restrictions on the dimensions, in particular the cross

sectional areas, of the mandrel and the housing. The mandrel and the housing are made of steel, with an elasticity modulus of $E_m = E_h = 2.1 \times 10^5$ N/mm². The outer diameter of the housing is restricted to a maximum of

5 181 mm. The inner diameter of a cylindrical mandrel is restricted to a minimum of 63.5 mm.

The bearings that are used are thrust bearings have an inner (shaft) diameter of 110 mm, outer (housing) diameter of 145 mm and axial length of 25 mm. The

10 stiffness of all the bearings, as obtained from the bearing manufacturer, is $K_{bearing} = 4.11 \times 10^6$ N/mm, defined as the ratio of the axial force applied to the bearing and the axial shortening of the bearing resulting from the axial force applied to the bearing. The mandrel

15 outer diameter along the sections of the mandrel is 100 mm such that the bearings can be fit and centred in the tool. The housing inner diameter along the sections of the housing is 150 mm. The section lengths (L_1, L_2, L_3, etc.) along the housing and mandrel required for the

20 bearings, spring and fixtures/fittings to the mandrel and housing is 75 mm.

In order to be able to transfer a load of 500 kN, per bearing stage a disc spring is provided having a thickness of 20 mm and a stiffness

25 $K_{spring} = 1.35 \times 10^6$ N/mm.

The stiffness K_h of each 75-mm section along the housing is given by:

$$K_h = \frac{E_h A_h}{L}$$

$$= \frac{2.1 \times 10^5 \times \frac{\pi}{4}\left(181^2 - 150^2\right)}{75} = 22.6 \times 10^6 \text{ N/mm}^2$$

The stiffness K_m of each 75-mm sections along the mandrel is given by:

$$K_m = \frac{E_m A_m}{L}$$

$$= \frac{2.1 \times 10^5 \times \frac{\pi}{4}\left(100^2 - 63.5^2\right)}{75} = 13.1 \times 10^6 \text{ N/mm}^2$$

At the first bearing stage two disc springs are mounted in series, which more practically halves the stiffness. The combined stiffness K_1 of the two springs and bearing in series at the first bearing stage is given by:

$$K_1 = \frac{\frac{1}{2}S_{spring} \times S_{bearing}}{\frac{1}{2}S_{spring} + S_{bearing}}$$

$$= \frac{\frac{1}{2} \times 1.35 \times 10^6 \times 4.11 \times 10^6}{\frac{1}{2} \times 1.35 \times 10^6 + 4.11 \times 10^6} = 0.58 \times 10^6 \text{ N/mm}^2$$

So that the relevant stiffness ratios K_1/K_h and K_1/K_m become:

$$\frac{K_1}{K_h} = \frac{0.58 \times 10^6}{22.6 \times 10^6} = 0.025,$$

$$\frac{K_1}{K_m} = \frac{0.58 \times 10^6}{13.1 \times 10^6} = 0.044$$

Using these values, the stiffness ratios for the other bearing stages can be calculated for which the load is divided proportionally over all the bearing stages:

$$\frac{K_2}{K_1} = \left[1 + 2\frac{K_1}{K_h} - 6\frac{K_1}{K_m}\right]^{-1} = [1 + 2 \times 0.025 - 6 \times 0.044]^{-1} = 1.27 \quad \Rightarrow \quad K_2 = 1.27 K_1$$

$$\frac{K_3}{K_1} = \left[1 + 6\frac{K_1}{K_h} - 10\frac{K_1}{K_m}\right]^{-1} = [1 + 6 \times 0.025 - 10 \times 0.044]^{-1} = 1.16 \quad \Rightarrow \quad K_3 = 1.16 K_1$$

$$\frac{K_4}{K_1} = \left[1 + 12\frac{K_1}{K_h} - 12\frac{K_1}{K_m}\right]^{-1} = [1 + 12 \times 0.025 - 12 \times 0.044]^{-1} = 1.29 \quad \Rightarrow \quad K_4 = 1.29 K_1$$

These higher values for the bearing stage stiffnesses can be achieved by mounting stiffer springs in series with the bearings than at stage 1. The relation for the combined stiffness K_k at a bearing stage k (with two springs in series) can be reformulated as follows:

$$K_k = \frac{\frac{1}{2}K_{spring,k} \times K_{bearing}}{\frac{1}{2}K_{spring,k} + K_{bearing}} \quad \Rightarrow \quad K_{spring,k} = 2\frac{K_{bearing} \times K_k}{K_{bearing} - K_k}$$

Now the required spring stiffness values for the bearing stages 2, 3, 4 are:

$$K_{spring,1} = 2\frac{K_{bearing} \times 1.27K_1}{K_{bearing} - 1.27K_1} = 2\frac{4.11 \times 1.27 \times 0.58}{4.11 - 1.27 \times 0.58} \times 10^6 = 1.796 \times 10^6 \ N/mm^2$$

$$K_{spring,2} = 2\frac{K_{bearing} \times 1.16K_1}{K_{bearing} - 1.16K_1} = 2\frac{4.11 \times 1.16 \times 0.58}{4.11 - 1.16 \times 0.58} \times 10^6 = 1.608 \times 10^6 \ N/mm^2$$

$$K_{spring,3} = 2\frac{K_{bearing} \times 1.29K_1}{K_{bearing} - 1.29K_1} = 2\frac{4.11 \times 1.29 \times 0.58}{4.11 - 1.29 \times 0.58} \times 10^6 = 1.830 \times 10^6 \ N/mm^2$$

In this practical example the differences in optimal stiffness values are relatively small. This is due the fact that by design the cross sectional areas of the mandrel and of the housing only differ a factor of about 2, which is favourable. In addition, by taking two springs in series at the first bearing stage, the values of the important ratios K_1/K_m and K_1/K_h could be restricted. With other design choices, the differences in optimal spring stiffness values could have become significantly larger.

Although for some applications, such as described above, the mandrel is preferably provided in the form of a tubular element, the invention also works for a solid mandrel, and even for housing and/or mandrel having a non-circular cross section such as a square cross section.

It is remarked that a bearing system as described herein can, in addition to its use in a bore hole tool assembly, advantageously be used in other fields of technology wherein relatively high axial loads are desired to be rotatably transferred from a housing to a mandrel or an axis, or vice versa. Examples of such fields include automotive applications, in particular heavy trucks, locomotives, drive shaft for marine vessels. This is also the case mutatis mutandis for the described method of designing the bore hole tool assembly.

Unpublished European patent application No. 02080230,
which serves as a priority application for the present
application and is herewith incorporated by reference,
describes detailed embodiments of a system for use in a
5   bore hole wherein the bore hole tool assembly in
accordance with the present invention can advantageously
be embodied.